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NUMERICAL INTEGRATOR FOR THE SOLUTION OF TWO HYBRID POINTS FOR STIFF AND OSCILLATORY DIFFERENTIAL EQUATIONS FOR FIRST ORDER INITIAL VALUE PROBLEMS OF ORDINARY DIFFERENTIAL EQUATIONS

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Abstract
This research paper examined the implementation of a hybrid integrator for the solution of Stiff and Oscillatory first order initial value problems of Ordinary Differential Equations. The hybrid integrator was derived by collocation and interpolation of the combination of power series and exponential function at some selected grid and off-grid points to generate a linear multistep method which is implemented in block form. The paper further investigated the properties of the block integrator and found it to be zero-stable, consistent and convergent. The region of absolute stability was also investigated. The method was tested on some notable numerical examples and the results was found to compete favourably with some existing methods.

Keywords: Stiff, Oscillatory, Collocation, Consistent, Convergent, Block integrator, Power Series, Interpolation
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Introduction

There are many processes in the field of sciences, engineering and management which involves rate of change of one or more quantities in relation to another, which may result into first order ordinary differential equations (ODEs). Usually mathematical models are developed to help in the understanding of physical phenomena, which often yields equations that contain some derivatives of an unknown function of one or several variables, such equations are called differential equations (D.E). Differential equations do not only arise in the physical sciences, but also in diverse fields of scientific disciplines that attempt to unveil the reality of the physical world. The development of numerical methods to obtain approximate solutions becomes necessary because most of these state equations do not have analytic solutions, with the advent of digital computers, application of numerical methods in solving real life problems has received attention of researchers.
This paper considered the numerical solution of stiff and oscillatory first order initial value problems (IVPs) in the form

$$y' = f(x, y), y(x_0) = y_0, x \in [a, b] \quad (1)$$

where x_0 is the initial point, y_0 is the solution at the initial point and f also assumed that (1) satisfy the existence and uniqueness theorem.
Fatunla, (1988) considers numerical integration scheme as schemes in which the approximate solutions are given by a linear combination of independent functions. Examples are Taylor series, Lie approach and Picard integration scheme. These methods generate the series of solutions whose convergence cannot be guaranteed, and when the series do converge, the rate of convergence may be rather too slow.

Statement of the Problem

Most numerical integrators developed for the solution of first order initial value problems has been reported to have setbacks such as computational burden, complication in writing computer

programs, lack of adequate stability characteristics, inefficiency in terms of accuracy and error term, low rate of convergence and resultant wastage of computer time (Awoyemi, Ademiluyi and Amuseghan, 2007). Also, LMMs implemented by the predictor-corrector mode have been found to be very expensive to implement and the predictors often have lower order of accuracy than the correctors especially when all the step and off step points are used for collocation and interpolation.

The application of hybrid methods in the LMMs to achieve reduction in the step number of the predictor-corrector mode is compounded by the need to develop predictors for the evaluation of the corrector at off step points making the approach even more tedious and time consuming (Lambert, 1973). The introduction of block methods to cushion the challenges associated with LMM implemented in the predictor-corrector mode has largely been concentrated in solving stiff initial value problems ordinary differential equations. The block method was found to be cost effective and give better approximation than predictor corrector. Most of the block methods developed in literature, the number of interpolation points must be equal to the order of the differential equations, the implication of this is that the method does not interpolate at all the grid points, hence, method of lower order are developed.

In view of the foregoing, this research is motivated by the need to address the setbacks associated with the existing methods by developing hybrid method that interpolate at all the grid points, using polynomial approximate solution. The implication of this is that method with higher order and better approximation are developed to handle stiff problems.

Objectives of the Study

The objectives of the study are to:

(i) develop a continuous scheme using finite power series and exponential function as basis function.

(ii) obtain discrete methods which are implemented in block form.

(iii) investigate the stability properties of the developed method.

(iv) write a code in MATLAB 8.5 to implement the method and test the method on some numerical examples.

Justification of the Study

The solution to stiff initial value problems is of course timely, because it occurs virtually in all human endeavors. Furthermore, the methods would be less expensive in terms of the number of function evaluation per step, more computationally reliable, highly efficient in terms of accuracy and error term, flexible in the change of step length, possesses better rate of convergence and very easy to program due to economy of computer time. It is also important to note that this research besides its enormous contribution to knowledge, will of course justify its importance in other disciplines like engineering and sciences that deals with ODEs. Researchers of Mathematics and other related disciplines will find it necessary in solving real life problems. The research will also serve as preliminary for further researches in related areas.

Scope of the Study

This research work would be restricted to numerical solutions of first order initial value problems of ordinary differential equations which will solve first order initial value problems. This is necessary for the fact that ODEs are quite a broad area of study. Boundary value problems are not considered unless they are converted to initial value problems. Methods of conversion are not considered in this research.

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Literature

Linear multistep method can be implemented in two basic forms namely predictor-corrector method and block method. The main disadvantage of the predictor-corrector method is the high cost of execution as subprograms are very difficult to code due to the separate methods needed to provide initial values (James and Adesanya, 2014). Block method was introduced to address the setbacks of the predictor-corrector.

In this research, our attention will be based on block method. Block method is formulated in terms of linear multistep methods (LMMs), It preserves the traditional advantage of one- step methods of being self - starting and permitting easy change of step length (Lambert, 1973). The block method produces numerical solutions with less computational efforts as compared to non-block method, this is because block method generates simultaneous solutions at all grid points (Majid, Suleiman and Omar, 2006). Scholars that worked on block methods for the solution of first order initial value problems include:

Akinfenwa, Yao and Jator, (2011), considered the development of a self-starting two step continuous hybrid block formulae (CHBF) using polynomial approximate solution. They used collocation and interpolation techniques to obtained an A (α)-stable method of order 7.

Akinfenwa, Jator and Yao (2011), considered the development of Backward differentiation formula with continuous coefficients for the solution of stiff ODEs using polynomial approximate solution. They used collocation and interpolation techniques to give L-stable method of order 8, implemented in block form.

Ibijola, Skwame and Kumleng, (2011) considered the development of continuous hybrid linear multistep methods n = 1. They considered multistep collocation procedure to obtain three, four and five discretes schemes. They used collocation and interpolation techniques to obtain an A-stable method of order 6, implemented in block form.

Adesanya, Odekunle and James (2012), considered the development of continuous hybrid method for the solution of first order ODEs using polynomial approximate solution. They used collocation and interpolation techniques to give method of order 7, implemented in block form.

Fotta, Alabi and Abdulqadir (2015), considered the development of hybrid block method for the solution of first order IVPs of ODEs using an approximate solution that combine power series and exponential function. They used collocation and interpolation techniques to obtain method of order 3.

Ra'ft, Zurni and Kaboye (2016), considered the development of hybrid block method for the solution of first order ODEs using polynomial approximate solution. They used collocation and interpolation techniques to obtain a method of order 4.

Rufai, Duromola and Ganiyu (2016), considered the development of hybrid block method for solving general first order ODEs using Chebyshev polynomial as approximate solution. They used collocation and interpolation techniques to obtain an A-stable method of order 4.

In this paper, the development of hybrid block integrator is carried out using an approximate solution which is a combination of power series and exponential function. It should be noted that our approximate solution considered will be more of exponential function than the one proposed by the authors mentioned above.

Methodology

We considered an approximate solution that combines power series and exponential function of the form

$$y(x) = \sum_{i=0}^{1} a_i e^{i} + \sum_{i=2}^{4} a_i e^{-ix} \quad (2)$$

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where $a_i's$ are unknown parameters to be determined. The first derivative of (2) gives

$$y'(x) = \sum_{i=0}^{1} ia_i e^{i-1} - \sum_{i=2}^{4} ia_i e^{-ix}$$

(**3**)

Substituting (3) into (2) gives

$$f(x,y) = \sum_{i=0}^{1} ia_i e^{i-1} - \sum_{i=2}^{4} ia_i e^{-ix}$$

(4)

Interpolating (2) at $x = x_n$ and collocating (4) at $x = x_{n+s}, s = 0, \frac{1}{3}, \frac{2}{3}, 1$ gives a system of non-linear equation in the form

$$XA = U$$

(5)

where

$$X = \begin{bmatrix} 1 & x_n & e^{-2x_n} & e^{-3x_n} & e^{-4x_n} \\ 0 & 1 & -2e^{-2x_n} & -3e^{-3x_n} & -4e^{-4x_n} \\ 0 & 1 & -2e^{-2x_{n+\frac{1}{3}}} & -3e^{-3x_{n+\frac{1}{3}}} & -4e^{-4x_{n+\frac{1}{3}}} \\ 0 & 1 & -2e^{-2x_{n+\frac{2}{3}}} & -3e^{-3x_{n+\frac{2}{3}}} & -4e^{-4x_{n+\frac{2}{3}}} \\ 0 & 1 & -2e^{-2x_{n+1}} & -3e^{-3x_{n+1}} & -4e^{-4x_{n+1}} \end{bmatrix}$$

and $A = [a_0 \quad a_1 \quad a_2 \quad a_3 \quad a_4]^T, U = [y_n \quad f_n \quad f_{n+\frac{1}{3}} \quad f_{n+\frac{2}{3}} \quad f_{n+1}]^T$. Solving (5) for the $a_i's$ and substituting back into (4) gives a linear multistep method of the form

$$y(t) = \alpha_0(t) y_n + h(\beta_0 f_n + \beta_{\frac{1}{3}} f_{n+\frac{1}{3}} + \beta_{\frac{2}{3}} f_{n+\frac{2}{3}} + \beta_1 f_{n+1}) \tag{6}$$

where $t = \frac{x - x_n}{h}$, $f_{n+j} = f(x_n + jh, y(x_n + jh))$

$$\begin{aligned} \alpha_0 &= 1 \\ \beta_0 &= -\frac{1}{8} h(9t^4 - 24t^3 + 22t^2 - 8t), \\ \beta_{\frac{1}{3}} &= \frac{3}{8} h(9t^4 - 20t^3 + 12t^2), \\ \beta_{\frac{2}{3}} &= -\frac{3}{8} h(9t^4 - 16t^3 + 6t^2), \\ \beta_1 &= \frac{1}{8} h(9t^4 - 12t^3 + 4t^2). \end{aligned} \tag{7}$$

evaluating (7) at $t = 0, \frac{1}{3}, \frac{2}{3}, 1$and writing in block form gives a discrete block formula of the form

$$A^{(0)} Y_m = e y_n + h d f(y_n) + h b F(Y_m) \tag{8}$$

where

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$$Y_m = [y_{n+\frac{1}{3}} \quad y_{n+\frac{2}{3}} \quad y_{n+1}]^T,$$

$$y_n = [y_{n-\frac{1}{3}} \quad y_{n-\frac{2}{3}} \quad y_n]^T,$$

$$F(Y_m) = [f_{n+\frac{1}{3}} \quad f_{n+\frac{2}{3}} \quad f_{n+1}]^T,$$

$$f(y_n) = [f_{n-\frac{1}{3}} \quad f_{n-\frac{2}{3}} \quad f_{n-1}]^T$$

$$A^{(0)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, e = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}, d = \begin{bmatrix} 0 & 0 & \frac{1}{8} \\ 0 & 0 & \frac{1}{9} \\ 0 & 0 & \frac{1}{8} \end{bmatrix}, b = \begin{bmatrix} \frac{19}{72} & -\frac{5}{72} & \frac{1}{72} \\ \frac{4}{9} & \frac{1}{9} & 0 \\ \frac{3}{8} & \frac{3}{8} & \frac{1}{8} \end{bmatrix}$$

Analysis of Basic Properties

Order of the block method

Let the linear operator $L\{y(x):h\}$ associated with the block integrator (8) be defined as

$$L\{y(x):h\} = A^{(0)}Y_m - ey_n + hdf(y_n) + hbF(Y_m) \tag{9}$$

Expanding using Taylor series and comparing the coefficients of h gives

$$L\{y(x):h\} = C_0 y(x) + C_1 hy'(x) + ... + C_p h^p y^p(x) + C_{p+1} h^{p+1} y^{p+1}(x) + ... \tag{10}$$

Order of Block Method

The linear operator L and associated block method are said to be of order p if $c_0 = c_1 = c_2 = ... = c_p = 0, c_{p+1} \neq 0. c_{p+1}$ is called the error constant and implies that the truncation error is given by

$$t_{n+k} = C_{p+1} h^{p+1} y^{p+1}(x) + O(h^{p+2}) \tag{11}$$

For our method

$$L\{y(x):h\} = \left[\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_{n+\frac{1}{3}} \\ y_{n+\frac{2}{3}} \\ y_{n+1} \end{bmatrix} - \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_{n-\frac{1}{3}} \\ y_{n-\frac{2}{3}} \\ y_n \end{bmatrix} - h \begin{bmatrix} \frac{1}{8} & \frac{19}{72} & -\frac{5}{72} & \frac{1}{72} \\ \frac{1}{9} & \frac{4}{9} & \frac{1}{9} & 0 \\ \frac{1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{1}{8} \end{bmatrix} \begin{bmatrix} f_n \\ f_{n+\frac{1}{3}} \\ f_{n+\frac{2}{3}} \\ f_{n+1} \end{bmatrix} \right] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{12}$$

Expanding (12) in Taylor series, we find that

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$$\begin{bmatrix} \sum_{j=0}^{\infty} \frac{(\frac{1}{3}h)^j}{j!} y'_n - y_n - \frac{1h}{8} y'_n - \sum_{j=0}^{\infty} \frac{h^{j+1}}{j!} y_n{}^{j+1} \left\{ \frac{19}{72}(\frac{1}{3})^j - \frac{5}{72}(\frac{2}{3})^j + \frac{1}{72}(1)^j \right\} \\ \sum_{j=0}^{\infty} \frac{(\frac{2h}{3})^j}{j!} y'_n - y_n - \frac{1h}{9} y'_n - \sum_{j=0}^{\infty} \frac{h^{j+1}}{j!} y_n{}^{j+1} \left\{ \frac{4}{9}(\frac{1}{3})^j - \frac{1}{9}(\frac{2}{3})^j + 0(1)^j \right\} \\ \sum_{j=0}^{\infty} \frac{(h)^j}{j!} y'_n - y_n - \frac{1h}{8} y'_n - \sum_{j=0}^{\infty} \frac{h^{j+1}}{j!} y_n{}^{j+1} \left\{ \frac{3}{8}(\frac{1}{3})^j + \frac{3}{8}(\frac{2}{3})^j + \frac{1}{8}(1)^j \right\} \end{bmatrix} \quad (13)$$

Hence, $c_0 = c_1 = c_2 = c_3 = c_4 = 0, c_5 \neq 0, \text{LTE} = [\text{-}1.0860(\text{-}04), \text{-}4.5725(\text{-}05), \text{-}1.5432(\text{-}4)]^T$ .The hybrid block integrator is of order 4.

Zero - Stability

The block integrator (8) is said to be zero stable, if the roots z_s, s = 1, 2,..., k of the first characteristics polynomial $\rho(z)$ defined by $\rho(z) = \det(zA^{(0)} - e)$ satisfies $|z_s| \leq 1$and if every root with modulus , $|z_s| = 1$ have multiplicity not exceeding the order of the differential equation. Moreover as $h \to 0, \rho(z) = z^{r-\mu}(z-1)^{\mu}$, where μ is the order of the differential equation, r is the order of the matrices A(0) and e.

$$\rho(z) = \left| z \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \right| = 0$$

$\rho(z) = z^3 - z^2 = z^2(z-1) = 0 \Rightarrow z_1 = z_2 = 0.z_3 = 1.$Hence our integrator is zero - stable.

Consistency

The block integrator (8) is consistent since it has order greater than one. From the above analysis, it is obvious that our integrator is consistent.

Convergence

The block integrator is convergent by the consequence of Dahlquist theorem below.

Theorem 1: (Dalhquist, 1956)

The necessary and sufficient conditions for a linear multistep method to be convergent are that it be consistent and zero stable.

Region of Absolute Stability.

Region of absolute stability is a region in the complex z plane, where $z = \lambda h$. It is defined as those values of z such that the numerical solution of $y' = -\lambda y$ satisfies $j_j \to 0$ as $j \to \infty$ for any initial condition.

To determine the absolute stability region of the new block Integrator, we adopt the boundary locus method. This is achieved by substituting the test equation

$$y' = -\lambda y \quad (14)$$

into the block formula (8).This gives

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$$A^{(0)}Y_m(w) = ey_n(w) - h\lambda Dy_n(w) - h\lambda BY_m(w) \tag{15}$$

Thus,

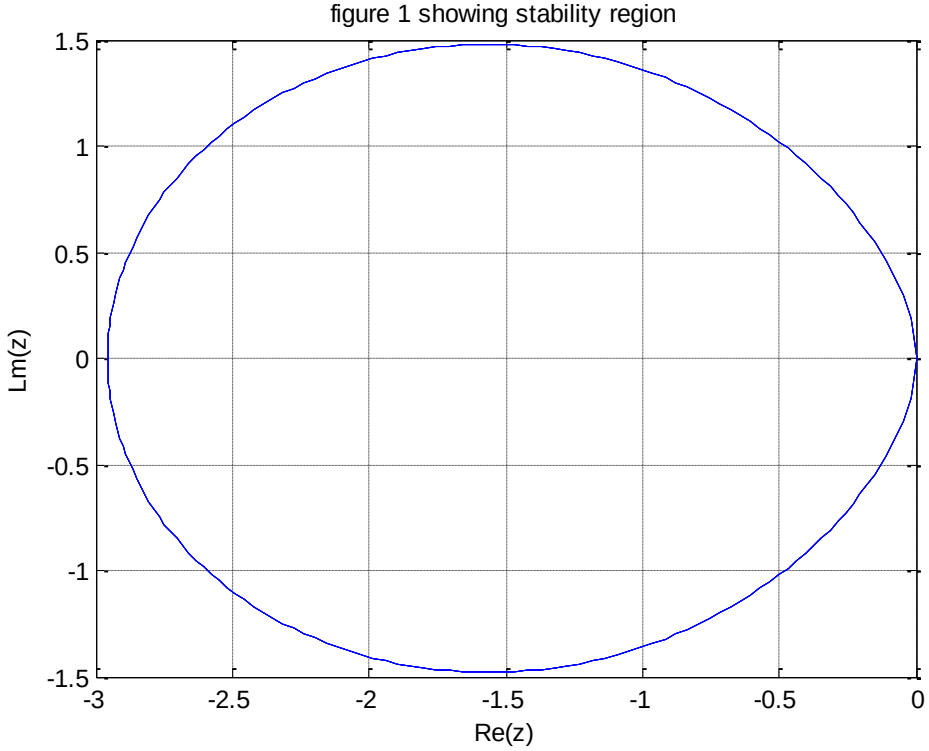
$$\bar{h}(w) = -\left(\frac{A^{(0)}Y_m(w) - ey_n(w)}{Dy_n(w) + BY_m(w)}\right) \tag{16}$$

Since $\bar{h}$ is given by $\bar{h} = \lambda h$ and $w = e^{i\theta}$. Equation (16) is our characteristics or stability polynomial for our integrator, equation (16) is given by

$$\bar{h}(w) = \left(-\frac{1}{108}w^3 - \frac{1}{108}w^2\right)h^3 + \left(\frac{11}{108}w^3 - \frac{11}{108}w^2\right)h^2 + \left(-\frac{1}{2}w^3 - \frac{1}{2}w^2\right)h + \left(w^3 - w^2\right)$$

The Region of Absolute Stability for the method is shown in figure 1 below.

figure 1 showing stability region



Numerical Examples

We shall evaluate the performance of the block integrator on some challenging stiff and oscillatory problems which have appeared in literature and compare the results with solutions from some methods of similar derivation. The numerical results are obtained using MATLAB.

The following notations shall be used in the tables below.

ERR - |Exact solution - Computed solution| (Error in new method)
ERS - Error in Sunday, et al., (2014).
ERO - Error in Okunuga, et al., (2013).
ERA - Error in Adebayo and Umar, et al., (2013).

Problem 1

Consider the highly stiff ordinary differential equation:

$$y' = -10(y-1)^2, y(0) = 2$$

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Exact solution: $y(x) = 1 + \frac{1}{1+10x}$

This problem was earlier discussed by Lambert (1973), the newly derived hybrid block integrator is used for the integration of this problem within the interval $0 \le x \le 0.1$. Okunuga et al., (2013) solved this stiff problem by adopting a new 2 - point hybrid block method with step size ratio at r = 1. Sunday et al., (2014) also solved problem 1 by applying a self - starting block integrator.

Problem 2

Consider the Prothero - Robinson Oscillatory ODE.

$$y' = L(y - \sin x) + \cos x, L = -1, y(0) = 0.$$

Exact solution: $y(x) = \sin x$

Adebayo and Umar (2013) solved this problem by adopting a generalized rational approximation method via Pade approximants with step number k = 6, r = 1. In a similar manner Sunday et al., (2014) solved problem 2 by applying a self - starting block integrator.

Table 1: Showing the results for stiff Problem 1

X	ERR	ERS	ERO
0.0100	2.879519e-011	3.414671e-006	2.07e-03
0.0200	4.097789e-011	2.749635e-006	3.30e-03
0.0300	4.514411e-011	1.342943e-005	3.21e-03
0.0400	4.539880e-011	9.090648e-005	6.30e-03
0.0500	4.378098e-011	7.969685e-005	8.53e-03
0.0600	4.132628e-011	6.994886e-005	9.00e-03
0.0700	3.856981e-011	6.270048e-005	9.98e-03
0.0800	3.578560e-011	6.017101e-005	2.06e-02
0.0900	3.311063e-011	5.411308e-005	2.10e-02
0.1000	3.060774e-011	4.880978e-005	2.12e-02

Table 2: Showing the results for Prothero - Robinson Oscillatory Problem 2

X	ERR	ERS	ERA
0.1000	1.473405e-013	1.452952e-011	3.0e-11
0.2000	2.791656e-013	1.621117e-011	4.0e-11
0.3000	3.954059e-013	2.131013e-011	2.0e-10
0.4000	4.962142e-013	1.379910e-011	3.0e-10
0.5000	5.817569e-013	2.744084e-011	2.0e-10
0.6000	6.520340e-013	1.111424e-011	3.0e-10
0.7000	7.072121e-013	2.865663e-011	2.0e-10
0.8000	7.477352e-013	1.921784e-010	3.0e-10
0.9000	7.736034e-013	1.239202e-010	4.0e-10
1.0000	7.857048e-013	1.471102e-010	4.0e-10

RESULTS

We have considered two numerical examples to test the efficiency of our method. The first problem (which is stiff) was solved by Okunuga, et al.,(2013), where they both applied 2-point block method with step-size ratio at r = 1, while the second problem (which is oscillatory) was solved by Adebayo and Umar (2013) where they adopted generalized rational approximation method via Pade approximants with step number k = 6. We solved the two problems using the new hybrid block integrator developed. Tables 1 and 2 above showed that the hybrid block integrator gives better

results than the existing methods. It should be noted that the method performs better when the step-size is chosen within the stability interval. The problems are first order ODEs; hence, the method performs better when the step size is within the stability interval.

CONCLUSION

In this paper, we have presented hybrid block integrator for the solution of stiff and oscillatory first order initial value problems in ordinary differential equations. Our method was found to be zero stable, consistent and convergent. The numerical results show that our method is computationally reliable and gave better accuracy than the existing methods.

RECOMMENDATIONS

The paper developed Numerical integrator with hybrid points for the numerical solution of stiff and oscillatory differential equations for first order initial value problems of ordinary differential equations. We recommend that further research work should be carried out using fourth order initial value problems of ordinary differential equations.

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