ISSN: 4120 – 5671
Volume 12, Number 1, 2026
Advanced Journal of Physical Science Research

Journal Homepage: www.bwjournal.org

USING NEWTON-RAPHSON'S METHOD TO CALCULATE THE APPROXIMATE SOLUTIONS OF THE VARIABLES OF A NON-LINEAR MODEL EQUATION FOR CHOLERA DISEASE AFTER SOME ITERATIONS.

Ochi P.O.[1], Jeremiah Timothy[2] and Timothy Ado Shamaki[3]
[1]Department of Mathematics, Nile University of Nigeria, Abuja, Nigeria.
[2]Department of Mathematics, Federal College of Education, Yola, P.M.B 2024, Yola, Nigeria. [3]Department of Applied Mathematics, Federal university of Technology, Babura, Jigawa State, Nigeria.

***Corresponding Author:** onyema141@gmail.com*

ABSTRACT

To solve a nonlinear equation, the Newton-Raphson technique employs the idea of iterative approximation. Every iteration is used to improve the original guess value for the answer and bring it closer to the real solution. By calculating the derivative of the nonlinear equation at that point, the iterations are based on linearization around the current guess. The guess is updated using the linear approximation by deducting the linearization's value from the prior guess. Until the precise answer is found, this procedure is repeated. In order to solve the non-linear equations, an iterative technique is a mathematical procedure that is utilized as a starting value to build a series of improving approximation solutions. This work aims to employ the Newton-Raphson approach to ascertain the convergence (approximate solution in variables) of the equations of a nonlinear infectious disease model after multiple iterations using Matlab. After that, an Excel program is used to plot the iteration graph.

Key-word: ***Infectious disease, Non-linear equation, Newton-Raphson Method, Convergence of a Solution, Matlab, Microsoft Excel, Approximate solution, exact solution.***

INTRODUCTION

Numerical analysis is a branch of mathematics that provides tools and methods for solving mathematical problems (Manishkumar & Vinay, 2021). Numerical analysis is the study of algorithms that use numerical approximation for the problems of mathematical analysis (Manishkumar & Vinay, 2021). Finding the solution of a set of non–linear equation has been a problem for past years. Newton Raphson method is considered as an easy way to find the convergence of nonlinear equations. The goal of the field of numerical analysis is to design and analyze the techniques to give approximate but accurate solutions to difficult problems. Many mathematical models of engineering, economics, physics, chemistry, science and other disciplines with different types of non-linear equations (Manishkumar & Vinay, 2021). In recent time, several scientists and engineers have been focused on solving non-linear equations both numerically and analytically. Solving nonlinear equations is one of the important research areas in numerical analysis but finding the exact solutions of nonlinear equations is a difficult task. In several occasions, it may not be easy to get the exact solutions. Hence, numerical methods are helpful to find the approximate solutions. One of the most frequently occurring problems in scientific work is to find the root of non-linear equations $F(y) = 0$. We always assume that $f(y)$ is continuously differentiable real-valued function of a real variable y. We focused on obtaining the root of nonlinear algebraic equations involving multiple variables using Newton-Raphson method.

An equation is said to be nonlinear when it involves terms of degree higher than 1 in the unknown quantity (Louis et al., 2005). These terms may be polynomial or capable of being broken down into

Taylor series of degrees higher than 1(Louis et al., 2005). If the nonlinear equations cannot be solved analytically, then the solutions of the equations must be approached using iterative methods.

Newton-Raphson's method allows us to approximate the solution of a function, which is the point where the function crosses the x-axis. The following should be noted when you use Newton-Raphson's method:

- The function must be in the form $f(y) = 0$.
- The more approximations we take, the closer we will get to the actual solution.
- For each approximation, we have to use our result from the previous approximation.
- The Newton-Raphson's method is

$$y^{(k+1)} = y^{(k)} - \frac{f\left(y^{(k)}\right)}{f'\left(y^{(k)}\right)}$$

where $x^{(k)}$ is the starting approximation and $y^{(k+1)}$ is the next approximation. Once $y^{(k)}$ and $y^{(k+1)}$ are exactly equal or equal to a specified number of decimal places, we stop further iteration(s).

Newton-Raphson's algorithm and its variants have been used for over 250 years to solve implicit nonlinear equations (Casella & Bernhard, 2021). The algorithm is iterative and the convergence to the desired solution crucially depends on the choice of the initial guess for the unknowns of the problem. Once the result of the iterations is close enough to the solution, under mild regularity conditions and under the assumption of non-singular Jacobian, the algorithm converges to the solution in a super-linear fashion.

In general, it may not be easy to obtain an initial guess close enough to the solution to ensure that the asymptotic convergence result is obtained after a small number of iterations (Casella & Bernhard, 2021). In fact, if the initial guess is sufficiently far from the sought-after solution, Newton-Raphson's algorithm may not converge at all to it.

The objective of the study is to application of Newton–Raphson's method to determine the convergence (the approximate solutions of the variables) of a non–linear infectious disease model equations $F\left(y^{(k)}\right) = f_i\left({y_1}^{(k)}, {y_2}^{(k)}, \cdot \quad \cdot \quad \cdot, {y_n}^{(k)}\right) = 0$ in many variables after performing some iterations.

APPLICATION OF NEWTON–RAPHSON'S METHOD

The Newton-Raphson method is also called the Newton method. This method is very strong and widely used for solving equations using numbers. It works by using a simple idea of making a straight-line approximation. It usually gets closer to the correct answer much quicker than other methods that only go at a steady pace. The Newton method is a type of repeated process, where it keeps trying to get better and better guesses for the root of the equation. Along with calculating the value of the function at each guess, it also finds the slope of the function. It is based on the idea that the next guess for the root is where the line that touches the function at that point crosses the x-axis. The values of the function and its slope are calculated each time, and the process is repeated until the answer becomes really close. This method is used to find approximate solutions to equations involving polynomials with one variable. It can also be used to solve systems of equations that are not linear. Instead of just computing $f\left(y^{(k)}\right)$ the derivative $f'\left(y^{(k)}\right)$ also calculated. It is assumed that the next estimate of the root (solution), is where the tangent crosses the x axis. The values of $f\left(y^{(k)}\right)$ and $f'\left(y^{(k)}\right)$ are calculated and the process repeated until it converges.

The method is used to find the approximate zeros of polynomial functions with one variable. It can also be used to solve nonlinear system of equations. A system of nonlinear equations is a set of equations as the following:

$$F\left(y^{(k)}\right) = f_i\left({y_1}^{(k)}, {y_2}^{(k)}, \cdot \quad \cdot \quad \cdot, {y_n}^{(k)}\right) = 0, \qquad (1)$$

where $(y_1, y_2, y_3, \cdots, y_n)^T \in \mathbb{R}^n$ and $f_i : \mathbb{R}^n \to \mathbb{R}$ is a nonlinear real function, $i = 1, 2, 3, \cdots, n$.

Multiple–dimensional Newton-Raphson's method

The function of n variables $y^{(k)} = \left(y_1{}^{(k)}, y_2{}^{(k)}, \cdots, y_n{}^{(k)}\right)$ is given by:

$$F\left(y^{(k)}\right) = \begin{bmatrix} f_1\left(y_1{}^{(k)}, y_2{}^{(k)}, y_3{}^{(k)}, y_4{}^{(k)}, y_5{}^{(k)}, y_6{}^{(k)}, \cdots, y_n{}^{(k)}\right) \\ f_2\left(y_1{}^{(k)}, y_2{}^{(k)}, y_3{}^{(k)}, y_4{}^{(k)}, y_5{}^{(k)}, y_6{}^{(k)}, \cdots, y_n{}^{(k)}\right) \\ f_3\left(y_1{}^{(k)}, y_2{}^{(k)}, y_3{}^{(k)}, y_4{}^{(k)}, y_5{}^{(k)}, y_6{}^{(k)}, \cdots, y_n{}^{(k)}\right) \\ \vdots \\ f_n\left(y_1{}^{(k)}, y_2{}^{(k)}, y_3{}^{(k)}, y_4{}^{(k)}, y_5{}^{(k)}, y_6{}^{(k)}, \cdots, y_n{}^{(k)}\right) \end{bmatrix}, \quad (2)$$

the root is $F(x) = 0$. With the multiple dimensions, the tangential line at will no longer be at point y rather it will have an n –dimensional tangential vector that is defined using the Jacobian matrix:

$$\mathrm{J}\left(y^{(k)}\right) = \left[\frac{\partial F}{\partial Y}\middle| y^{(k)}\right], \quad (3)$$

to define the tangent vector as $\mathrm{J}(y)$. Now we reformulate Newton-Raphson's method for a single equation

$$y^{(k+1)} = y^{(k)} - \frac{f\left(y^{(k)}\right)}{f'\left(y^{(k)}\right)}$$

can be re-written as

$$y^{(k+1)} = y^{(k)} - J\left(y^{(k)}\right)^{-1} F\left(y^{(k)}\right) \quad (4)$$

Literature review

Moheuddin, M. M., Uddin, M. J., & Kowsher, M. (2019). The major goal of this study is to determine the optimal approach for solving the nonlinear equation using iterative methods. The objective of this research is to determine the rate of convergence, the correct solution, and the level of errors in the methodologies. The researcher aims at comparing existing methods in order to find the most effective method for solving nonlinear equations. The researcher discussed four iterative methods, their rates of convergence, and how they compare to graphic representation. The researcher has found that Newton-Raphson Method is the most effective and precise method for solving non-linear equations.

Ebelechukwu, O. C. (2018). The aim of this research is to find the most appropriate method for solving nonlinear equations. Four methods for solving nonlinear equations were explained in this paper. The objective of this research is to find the best outcome for using numerical methods to solve nonlinear equations. The researcher has compared the various approaches to determine which solution (or methods) is best for the particular problem. The research has found that Newton-Raphson Method is the most effective method for finding the roots of non-linear equations, because it converges to the roots of the non-linear equation is faster than the other three ways, depending on the results obtained from the four methods. In comparison to the other three methods, which take a long time to converge, it converges after a few iterations.

Hasan, A. (2016) provide a numerical analysis of some iterative methods for solving nonlinear equations in this research. The goal of the research is to compare the rates of performance (convergence) of Bisection, Newton-Raphson, and Secant as root-finding methods. The Bisection method converges at the 47th iteration, whereas the Newton-Raphson and Secant methods converge at the 4th and 5th iterations, respectively, to the exact root of 0.36042170296032 with the same error level. The Newton approach, in comparison to the Secant method, has less iterations. The Secant approach was subsequently shown to be the most successful of the three ways considered. Numerical experiments are used to illustrate that the secant approach is more efficient than other methods. Researcher concluded that the secant method is formally the most effective of the Newton method.

In general, the solution of a non-linear equation system by using the Newton method and its variants requires an initial value and the derivative of the non-linear equations involved in the system. The problem of solving the system of non-linear equations can be viewed as a matter of multi-objective optimization. Every equation in the system states an objective function that aims to minimize the difference between the right and left terms of the corresponding equation (Grosan & Abraham, 2008).

A new algorithm based on the method of Adomian decomposition convergence basis for solving functional equations is presented. This algorithm can account for all the real answers of a system if a suitable and primary approximation is chosen (Hosseini & Kafash, 2010).

An iterative method for solving the problem of the non-linear equation numerically is suggested. A mathematical proof supports the proposed iteration through the n-dimensional Taylor expansion (Montazeri et al., 2012).

Then, a new algorithm is proposed for the solutions of systems of non-linear equations which use a combination of the gradient and the Newton methods. A novel dynamic combinatory is developed to determine the contribution of the methods in the combination. The numerical results prove that the proposed combination algorithm is generally more robust and efficient than other methods on some important and difficult problems (Taheri & Mammadov, 2012).

Mathematical formulation of multi–dimensional variables from Newton-Raphson's one-dimensional variable

In numerical analysis, the Newton-Raphson Method is a method for finding successively better approximations to the roots (or zeros) of a real-valued function.

The general form of the Newton-Raphson Method can only be used to solve nonlinear equations with a single variable. Therefore, in order to solve a system of nonlinear equations involving multiple variables, we need to alter the general form of the Newton-Raphson Method.

A system of nonlinear equations is a set of equations as the following:

$$\left.\begin{array}{c} f_1\left(y_1^{(k)}, y_2^{(k)}, y_3^{(k)}, y_4^{(k)}, y_5^{(k)}, y_6^{(k)}, \cdot\ \cdot\ \cdot, y_n^{(k)}\right) = 0 \\ f_2\left(y_1^{(k)}, y_2^{(k)}, y_3^{(k)}, y_4^{(k)}, y_5^{(k)}, y_6^{(k)}, \cdot\ \cdot\ \cdot, y_n^{(k)}\right) = 0 \\ f_3\left(y_1^{(k)}, y_2^{(k)}, y_3^{(k)}, y_4^{(k)}, y_5^{(k)}, y_6^{(k)}, \cdot\ \cdot\ \cdot, y_n^{(k)}\right) = 0 \\ \cdot \\ \cdot \\ \cdot \\ f_n\left(y_1^{(k)}, y_2^{(k)}, y_3^{(k)}, y_4^{(k)}, y_5^{(k)}, y_6^{(k)}, \cdot\ \cdot\ \cdot, y_n^{(k)}\right) = 0 \end{array}\right\} \tag{5}$$

where $\left(y_1^{(k)}, y_2^{(k)}, \cdot\ \cdot\ \cdot, y_n^{(k)}\right)^T \in \mathbb{R}^n$ and $f_i : \mathbb{R}^n \to \mathbb{R}$ is a nonlinear real function, $i = 1, 2, 3, \cdot\ \cdot\ \cdot, n.$

We express a system of nonlinear equations as a vector from $F\left(y^{(k)}\right) = 0$, i.e.

$$F\left(y^{(k)}\right) = \begin{bmatrix} f_1\left(y_1^{(k)}, y_2^{(k)}, y_3^{(k)}, y_4^{(k)}, y_5^{(k)}, \cdot\ \cdot\ \cdot, y_n^{(k)}\right) \\ f_2\left(y_1^{(k)}, y_2^{(k)}, y_3^{(k)}, y_4^{(k)}, y_5^{(k)}, \cdot\ \cdot\ \cdot, y_n^{(k)}\right) \\ f_3\left(y_1^{(k)}, y_2^{(k)}, y_3^{(k)}, y_4^{(k)}, y_5^{(k)}, \cdot\ \cdot\ \cdot, y_n^{(k)}\right) \\ \cdot \\ \cdot \\ \cdot \\ f_n\left(y_1^{(k)}, y_2^{(k)}, y_3^{(k)}, y_4^{(k)}, y_5^{(k)}, \cdot\ \cdot\ \cdot, y_n^{(k)}\right) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix} \tag{6}$$

$$J\left(y^{(k)}\right) = \begin{pmatrix} \frac{\partial f_1(y^{(k)})}{\partial y_1} & \frac{\partial f_1(y^{(k)})}{\partial y_2} & \frac{\partial f_1(y^{(k)})}{\partial y_3} & \frac{\partial f_1(y^{(k)})}{\partial y_4} & \frac{\partial f_1(y^{(k)})}{\partial y_5} & \cdots & \frac{\partial f_1(y^{(k)})}{\partial y_n} \\ \frac{\partial f_2(y^{(k)})}{\partial y_1} & \frac{\partial f_2(y^{(k)})}{\partial y_2} & \frac{\partial f_2(y^{(k)})}{\partial y_3} & \frac{\partial f_1(y^{(k)})}{\partial y_4} & \frac{\partial f_1(y^{(k)})}{\partial y_5} & \cdots & \frac{\partial f_2(y^{(k)})}{\partial y_n} \\ \frac{\partial f_3(y^{(k)})}{\partial y_1} & \frac{\partial f_3(y^{(k)})}{\partial y_2} & \frac{\partial f_3(y^{(k)})}{\partial y_3} & \frac{\partial f_1(y^{(k)})}{\partial y_4} & \frac{\partial f_1(y^{(k)})}{\partial y_5} & \cdots & \frac{\partial f_3(y^{(k)})}{\partial y_n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n(y^{(k)})}{\partial y_1} & \frac{\partial f_n(y^{(k)})}{\partial y_2} & \frac{\partial f_n(y^{(k)})}{\partial y_3} & \frac{\partial f_1(y^{(k)})}{\partial y_4} & \frac{\partial f_1(y^{(k)})}{\partial y_5} & \cdots & \frac{\partial f_n(y^{(k)})}{\partial y_n} \end{pmatrix} \quad (7)$$

$$\left|J\left(y^{(k)}\right)\right| = \begin{vmatrix} \frac{\partial f_1(y^{(k)})}{\partial y_1} & \frac{\partial f_1(y^{(k)})}{\partial y_2} & \frac{\partial f_1(y^{(k)})}{\partial y_3} & \frac{\partial f_1(y^{(k)})}{\partial y_4} & \frac{\partial f_1(y^{(k)})}{\partial y_5} & \cdots & \frac{\partial f_1(y^{(k)})}{\partial y_n} \\ \frac{\partial f_2(y^{(k)})}{\partial y_1} & \frac{\partial f_2(y^{(k)})}{\partial y_2} & \frac{\partial f_2(y^{(k)})}{\partial y_3} & \frac{\partial f_1(y^{(k)})}{\partial y_4} & \frac{\partial f_1(y^{(k)})}{\partial y_5} & \cdots & \frac{\partial f_2(y^{(k)})}{\partial y_n} \\ \frac{\partial f_3(y^{(k)})}{\partial y_1} & \frac{\partial f_3(y^{(k)})}{\partial y_2} & \frac{\partial f_3(y^{(k)})}{\partial y_3} & \frac{\partial f_1(y^{(k)})}{\partial y_4} & \frac{\partial f_1(y^{(k)})}{\partial y_5} & \cdots & \frac{\partial f_3(y^{(k)})}{\partial y_n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n(y^{(k)})}{\partial y_1} & \frac{\partial f_n(y^{(k)})}{\partial y_2} & \frac{\partial f_n(y^{(k)})}{\partial y_3} & \frac{\partial f_1(y^{(k)})}{\partial y_4} & \frac{\partial f_1(y^{(k)})}{\partial y_5} & \cdots & \frac{\partial f_n(y^{(k)})}{\partial y_n} \end{vmatrix} \quad (8)$$

where $\left({y_1}^{(k)}, {y_2}^{(k)}, \cdots, {y_n}^{(k)}\right)^T \in \mathbb{R}^n$, $F : \mathbb{R}^n \to \mathbb{R}^n$, $f_i : \mathbb{R}^n \to \mathbb{R}$ and the Newton-Raphson Method as a matrix with a corresponding vector,

$y^{(k+1)} = y^{(k)} - J\left[y^{(k)}\right]^{-1} F\left[y^{(k)}\right]$, $y \in \mathbb{R}^n$, $k = 0,1,2,3,\cdots,n$.

$$\begin{bmatrix} {y_1}^{(k)} \\ {y_2}^{(k)} \\ {y_3}^{(k)} \\ \vdots \\ {y_n}^{(k)} \end{bmatrix} = \begin{bmatrix} {y_1}^{(k-1)} \\ {y_2}^{(k-1)} \\ {y_3}^{(k-1)} \\ \vdots \\ {y_n}^{(k-1)} \end{bmatrix} - \begin{bmatrix} \frac{\partial f_1(y^{(k-1)})}{\partial y_1} & \frac{\partial f_1(y^{(k-1)})}{\partial y_2} & \frac{\partial f_1(y^{(k-1)})}{\partial y_3} & \cdots & \frac{\partial f_1(y^{(k-1)})}{\partial y_n} \\ \frac{\partial f_2(y^{(k-1)})}{\partial y_1} & \frac{\partial f_2(y^{(k-1)})}{\partial y_2} & \frac{\partial f_2(y^{(k-1)})}{\partial y_3} & \cdots & \frac{\partial f_2(y^{(k-1)})}{\partial y_n} \\ \frac{\partial f_3(y^{(k-1)})}{\partial y_1} & \frac{\partial f_3(y^{(k-1)})}{\partial y_2} & \frac{\partial f_3(y^{(k-1)})}{\partial y_3} & \cdots & \frac{\partial f_3(y^{(k-1)})}{\partial y_n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n(y^{(k-1)})}{\partial y_1} & \frac{\partial f_n(y^{(k-1)})}{\partial y_2} & \frac{\partial f_n(y^{(k-1)})}{\partial y_3} & \cdots & \frac{\partial f_n(y^{(k-1)})}{\partial y_n} \end{bmatrix}^{-1} \begin{bmatrix} f_1(y^{(k-1)}) \\ f_2(y^{(k-1)}) \\ f_3(y^{(k-1)}) \\ \vdots \\ f_n(y^{(k-1)}) \end{bmatrix}$$

$$C(y) = \begin{bmatrix} {M_{11}}^{(k)} & {M_{12}}^{(k)} & {M_{13}}^{(k)} & {M_{14}}^{(k)} & {M_{15}}^{(k)} & \cdots & {M_{1n}}^{(k)} \\ {M_{21}}^{(k)} & {M_{22}}^{(k)} & {M_{23}}^{(k)} & {M_{24}}^{(k)} & {M_{25}}^{(k)} & \cdots & {M_{2n}}^{(k)} \\ {M_{31}}^{(k)} & {M_{32}}^{(k)} & {M_{33}}^{(k)} & {M_{34}}^{(k)} & {M_{35}}^{(k)} & \cdots & {M_{3n}}^{(k)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ {M_{n1}}^{(k)} & {M_{n2}}^{(k)} & {M_{n3}}^{(k)} & {M_{n4}}^{(k)} & {M_{n5}}^{(k)} & \cdots & {M_{nn}}^{(k)} \end{bmatrix} \quad (9)$$

where $C(y) = {M_{ij}}^{(k)}$ is the co-factor from the elements ${a_{ij}}^{(k)}$ of the matrix $J_k(y)$.

$$\mathrm{AdjJ}(y) = [\mathrm{C}(y)]^T = \begin{bmatrix} {M_{11}}^{(k)} & {M_{21}}^{(k)} & {M_{31}}^{(k)} & {M_{41}}^{(k)} & {M_{51}}^{(k)} & \cdots & {M_{n1}}^{(k)} \\ {M_{12}}^{(k)} & {M_{22}}^{(k)} & {M_{32}}^{(k)} & {M_{42}}^{(k)} & {M_{52}}^{(k)} & \cdots & {M_{n2}}^{(k)} \\ {M_{13}}^{(k)} & {M_{23}}^{(k)} & {M_{33}}^{(k)} & {M_{43}}^{(k)} & {M_{53}}^{(k)} & \cdots & {M_{n3}}^{(k)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ {M_{1n}}^{(k)} & {M_{2n}}^{(k)} & {M_{3n}}^{(k)} & {M_{4n}}^{(k)} & {M_{5n}}^{(k)} & \cdots & {M_{nn}}^{(k)} \end{bmatrix} \quad (10)$$

$$\mathrm{J}\left[y^{(k)}\right]^{-1} = \frac{\mathrm{AdjJ}(y)}{|J_k(y)|} \quad (11)$$

$$= \frac{1}{|J_k|}\begin{bmatrix} {M_{11}}^{(k)} & {M_{21}}^{(k)} & {M_{31}}^{(k)} & {M_{41}}^{(k)} & {M_{51}}^{(k)} & \cdots & {M_{n1}}^{(k)} \\ {M_{12}}^{(k)} & {M_{22}}^{(k)} & {M_{32}}^{(k)} & {M_{42}}^{(k)} & {M_{52}}^{(k)} & \cdots & {M_{n2}}^{(k)} \\ {M_{13}}^{(k)} & {M_{23}}^{(k)} & {M_{33}}^{(k)} & {M_{43}}^{(k)} & {M_{53}}^{(k)} & \cdots & {M_{n3}}^{(k)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ {M_{1n}}^{(k)} & {M_{2n}}^{(k)} & {M_{3n}}^{(k)} & {M_{4n}}^{(k)} & {M_{5n}}^{(k)} & \cdots & {M_{nn}}^{(k)} \end{bmatrix} \quad (12)$$

$$\begin{bmatrix} {y_1}^{(k+1)} \\ {y_2}^{(k+1)} \\ {y_3}^{(k+1)} \\ \vdots \\ {y_n}^{(k+1)} \end{bmatrix} = \begin{bmatrix} {y_1}^{(k)} \\ {y_2}^{(k)} \\ {y_3}^{(k)} \\ \vdots \\ {y_n}^{(k)} \end{bmatrix} - \frac{1}{|J_k(y)|}\begin{bmatrix} {M_{11}}^{(k)} & {M_{21}}^{(k)} & {M_{31}}^{(k)} & {M_{41}}^{(k)} & \cdots & {M_{n1}}^{(k)} \\ {M_{12}}^{(k)} & {M_{22}}^{(k)} & {M_{32}}^{(k)} & {M_{42}}^{(k)} & \cdots & {M_{n2}}^{(k)} \\ {M_{13}}^{(k)} & {M_{23}}^{(k)} & {M_{33}}^{(k)} & {M_{43}}^{(k)} & \cdots & {M_{n3}}^{(k)} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ {M_{1n}}^{(k)} & {M_{2n}}^{(k)} & {M_{3n}}^{(k)} & {M_{4n}}^{(k)} & \cdots & {M_{nn}}^{(k)} \end{bmatrix}\begin{bmatrix} {f_1}^{(k)} \\ {f_2}^{(k)} \\ {f_3}^{(k)} \\ \vdots \\ {f_n}^{(k)} \end{bmatrix} \quad (13)$$

We repeat this process until $y^{(k+1)}$ converges to y^*, thus y^* is a solution of $F(y) = 0$.

Application of multi–dimensional Newton–Raphson's method to an infectious disease model equations

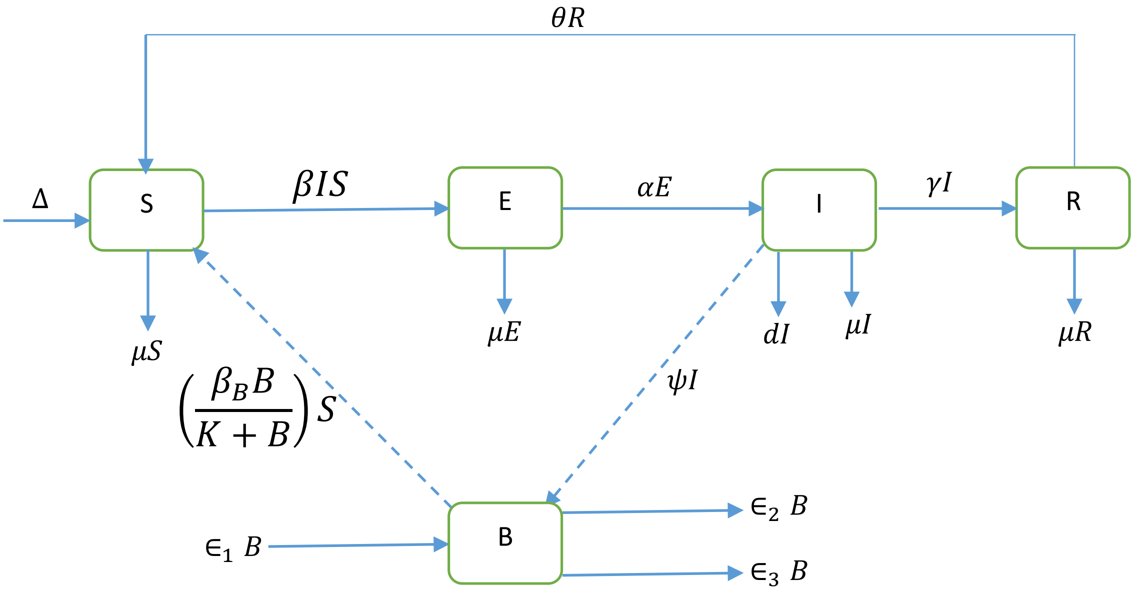

Figure 1: Flow diagram for the SEIRB Model

Model equations of cholera epidemic diseases

$$\left.\begin{aligned}\frac{dS}{dt} &= \Delta - \left(\beta I + \frac{\beta_B B}{K+B}\right)S + \theta R - \mu S\\ \frac{dE}{dt} &= \left(\beta I + \frac{\beta_B B}{K+B}\right)S - (\alpha+\mu)E\\ \frac{dI}{dt} &= \alpha E - (\gamma + d + \mu)I\\ \frac{dR}{dt} &= \gamma I - (\theta+\mu)R\\ \frac{dB}{dt} &= \psi I - (\epsilon_2 + \epsilon_3 - \epsilon_1)B\end{aligned}\right\} \quad (13)$$

Let $f_1 = \frac{dS}{dt} = \Delta - \left(\beta I + \frac{\beta_B B}{K+B}\right)S + \theta R - \mu S$

$\frac{\partial f_1}{\partial S} = -\left(\beta I + \frac{\beta_B B}{K+B} + \mu\right),\ \frac{\partial f_1}{\partial E} = 0,\ \frac{\partial f_1}{\partial I} = -\beta I,\ \frac{\partial f_1}{\partial R} = \theta,\ \frac{\partial f_1}{\partial B} = -\frac{K\beta_B}{(K+B)^2}$

$f_2 = \left(\beta I + \frac{\beta_B B}{K+B}\right)S - (\alpha+\mu)E$

$\frac{\partial f_2}{\partial S} = \left(\beta I + \frac{\beta_B B}{K+B}\right),\ \frac{\partial f_2}{\partial E} = -(\alpha+\mu),\ \frac{\partial f_2}{\partial I} = \beta S,\ \frac{\partial f_2}{\partial R} = 0,\ \frac{\partial f_2}{\partial B} = \frac{K\beta_B}{(K+B)^2}$

$f_3 = \alpha E - (\gamma + \omega + \mu)I$

$\frac{\partial f_3}{\partial S} = 0,\ \frac{\partial f_3}{\partial E} = \alpha,\ \frac{\partial f_3}{\partial I} = -(\gamma+\omega+\mu),\ \frac{\partial f_3}{\partial R} = 0,\ \frac{\partial f_3}{\partial B} = 0$

$\frac{\partial f_4}{\partial S} = 0,\ \frac{\partial f_4}{\partial E} = 0,\ \frac{\partial f_4}{\partial I} = \gamma,\ \frac{\partial f_4}{\partial R} = -(\theta+\mu),\ \frac{\partial f_4}{\partial S} = 0$

$\frac{\partial f_5}{\partial S} = 0,\ \frac{\partial f_5}{\partial E} = 0,\ \frac{\partial f_5}{\partial I} = \psi,\ \frac{\partial f_5}{\partial R} = 0,\ \frac{\partial f_5}{\partial S} = -(\epsilon_2 + \epsilon_3 - \epsilon_1)$

Therefore, substitute the partial derivatives into the Jacobian matrix.

$$J_k = \begin{pmatrix} \frac{\partial f_1}{\partial S} & \frac{\partial f_1}{\partial I} & \frac{\partial f_1}{\partial E} & \frac{\partial f_1}{\partial R} & \frac{\partial f_1}{\partial B}\\ \frac{\partial f_2}{\partial S} & \frac{\partial f_2}{\partial E} & \frac{\partial f_2}{\partial I} & \frac{\partial f_2}{\partial R} & \frac{\partial f_2}{\partial B}\\ \frac{\partial f_3}{\partial S} & \frac{\partial f_3}{\partial E} & \frac{\partial f_3}{\partial I} & \frac{\partial f_3}{\partial R} & \frac{\partial f_3}{\partial B}\\ \frac{\partial f_4}{\partial S} & \frac{\partial f_4}{\partial E} & \frac{\partial f_4}{\partial I} & \frac{\partial f_4}{\partial R} & \frac{\partial f_4}{\partial B}\\ \frac{\partial f_5}{\partial S} & \frac{\partial f_5}{\partial E} & \frac{\partial f_5}{\partial I} & \frac{\partial f_5}{\partial R} & \frac{\partial f_5}{\partial B} \end{pmatrix}_{y^{(k)}} \quad (14)$$

The Jacobian matrix f_1, f_2, f_3, f_4, f_5 evaluated at $y^{(k)}$

$$J_k = \begin{pmatrix} -\left(\beta I_k + \frac{\beta_B B}{K+B_k} + \mu\right) & 0 & -\beta I_k & \theta & -\frac{K\beta_B}{(K+B_k)^2}\\ \left(\beta I_k + \frac{\beta_B B}{K+B_k}\right) & -(\alpha+\mu) & \beta I_k & 0 & \frac{K\beta_B}{(K+B_k)^2}\\ 0 & \alpha & -(\gamma+\omega+\mu) & 0 & 0\\ 0 & 0 & \gamma & -(\theta+\mu) & 0\\ 0 & 0 & \psi & 0 & -(\epsilon_2+\epsilon_3-\epsilon_1) \end{pmatrix} \quad (15)$$

$$|J_k| = \begin{vmatrix} -\left(\beta I_k + \frac{\beta_B B}{K+B_k} + \mu\right) & 0 & -\beta I_k & \theta & -\frac{K\beta_B}{(K+B_k)^2}\\ \left(\beta I_k + \frac{\beta_B B}{K+B_k}\right) & -(\alpha+\mu) & \beta I_k & 0 & \frac{K\beta_B}{(K+B_k)^2}\\ 0 & \alpha & -(\gamma+\omega+\mu) & 0 & 0\\ 0 & 0 & \gamma & -(\theta+\mu) & 0\\ 0 & 0 & \psi & 0 & -(\epsilon_2+\epsilon_3-\epsilon_1) \end{vmatrix} \quad (16)$$

Where $x_1 = \left(\beta I_k + \frac{\beta_B B}{K+B_k} + \mu\right),\ x_2 = \left(\beta I_k + \frac{\beta_B B}{K+B_k}\right),\ x_3 = -(\alpha+\mu),\ x_4 = -(\gamma+\omega+\mu),$

$x_5 = -(\theta+\mu),\ x_6 = -(\epsilon_2+\epsilon_3-\epsilon_1),\ x_7 = \frac{K\beta_B}{(K+B_k)^2}$

$D_k = |J_k| = x_1 x_5[\alpha(\beta x_6 S_k + \psi x_7) - x_3 x_4 x_6] - \alpha x_2[\psi x_5 x_7 + x_6(\beta x_6 S_k - \theta\gamma)]$

Let C_k be the co-factor of the Jacobian matrix J_k.

ISSN: 4120 – 5671
Volume 12, Number 1, 2026
Advanced Journal of Physical Science Research

Journal Homepage: www.bwjournal.org

$C_{11} = x_3x_4x_5x_6 - \alpha x_5(\beta x_6 S_k + \psi x_7)$
$C_{12} = x_2x_4x_5x_6$
$C_{13} = \alpha x_2x_5x_6$
$C_{14} = \alpha\gamma x_2x_6$
$C_{15} = \alpha\psi x_2x_5$
$C_{21} = \alpha[\psi x_5x_7 + x_6(\beta x_5 S_k - \theta\gamma)]$
$C_{22} = x_1x_4x_5x_6$
$C_{23} = \alpha x_1x_5x_6$
$C_{24} = \alpha\gamma x_1x_6$
$C_{25} = \alpha\psi x_1x_5$
$C_{31} = \psi x_3x_5x_7 - x_3x_6(\beta x_5 - \theta\gamma)$
$C_{32} = x_1x_5(\beta x_6 S_k - \psi x_7) - x_2(\psi x_5x_7 + x_6(\beta x_5 S_k - \theta\gamma))$
$C_{33} = x_1x_3x_5x_6$
$C_{34} = \gamma x_1x_3x_6$
$C_{35} = \psi x_1x_3x_5$
$C_{41} = \theta[x_3x_4x_6 - \alpha(\beta x_6 S_k - \psi x_7)]$
$C_{42} = \theta x_2x_4x_6$
$C_{43} = \theta\alpha x_2x_6$
$C_{44} = x_1x_3x_4x_6 - \alpha(\beta x_6 S_k + \psi x_7)(x_2 - x_1)$
$C_{45} = \alpha\theta\psi x_2$
$C_{51} = x_7[\alpha\theta\gamma - x_3x_4x_5]$
$C_{52} = x_4x_5x_7(x_1 - x_2)$
$C_{53} = \alpha x_5x_7(x_1 - x_2)$
$C_{54} = \alpha\gamma x_7(x_1 - x_2)$
$C_{55} = x_1x_5(x_3x_4 - \alpha\beta S_k) + \alpha x_2(\beta x_5 S_k - \theta\gamma)$

$$C_k = \begin{pmatrix} x_3x_4x_5x_6 - \alpha x_5(\beta x_6 S_k + \psi x_7) & x_2x_4x_5x_6 & \alpha x_2x_5x_6 & \alpha\gamma x_2x_6 & \alpha\psi x_2x_5 \\ \alpha[\psi x_5x_7 + x_6(\beta x_5 S_k - \theta\gamma)] & x_1x_4x_5x_6 & \alpha x_1x_5x_6 & \alpha\gamma x_1x_6 & \alpha\psi x_1x_5 \\ \psi x_3x_5x_7 - x_3x_6(\beta x_5 - \theta\gamma) & C_{32} & x_1x_3x_5x_6 & \gamma x_1x_3x_6 & \psi x_1x_3x_5 \\ \theta[x_3x_4x_6 - \alpha(\beta x_6 S_k - \psi x_7)] & \theta x_2x_4x_6 & \theta\alpha x_2x_6 & C_{44} & \alpha\theta\psi x_2 \\ x_7[\alpha\theta\gamma - x_3x_4x_5] & C_{52} & C_{53} & C_{54} & C_{55} \end{pmatrix}$$

$C_k{}^T = Adj J_k$

$$= \begin{pmatrix} x_3x_4x_5x_6 - \alpha x_5(\beta x_6 S_k + \psi x_7) & C_{21} & C_{31} & C_{41} & x_7[\alpha\theta\gamma - x_3x_4x_5] \\ x_2x_4x_5x_6 & x_1x_4x_5x_6 & C_{32} & \theta x_2x_4x_6 & x_4x_5x_7(x_1 - x_2) \\ \alpha x_2x_5x_6 & \alpha x_1x_5x_6 & x_1x_3x_5x_6 & \theta\alpha x_2x_6 & \alpha x_5x_7(x_1 - x_2) \\ \alpha\gamma x_2x_6 & \alpha\gamma x_1x_6 & \gamma x_1x_3x_6 & C_{44} & \alpha\gamma x_7(x_1 - x_2) \\ \alpha\psi x_2x_5 & \alpha\psi x_1x_5 & \psi x_1x_3x_5 & \alpha\theta\psi x_2 & C_{55} \end{pmatrix}$$

$J_k{}^{-1} = \frac{Adj\, J_k}{|J_k|} = \frac{1}{|J_k|} Adj\, J_k$
$C_{11} = M_{11}{}^{(k)} = x_3x_4x_5x_6 - \alpha x_5(\beta x_6 S_k + \psi x_7)$
$C_{21} = M_{12}{}^{(k)} = \alpha[\psi x_5x_7 + x_6(\beta x_5 S_k - \theta\gamma)]$
$C_{31} = M_{13}{}^{(k)} = \psi x_3x_5x_7 - x_3x_6(\beta x_5 - \theta\gamma)$
$C_{41} = M_{14}{}^{(k)} = \theta[x_3x_4x_6 - \alpha(\beta x_6 S_k - \psi x_7)]$
$C_{51} = M_{15}{}^{(k)} = x_7[\alpha\theta\gamma - x_3x_4x_5]$

$$J_k{}^{-1} = \frac{1}{|J_k|}\begin{pmatrix} C_{11}{}^{(k)} & C_{21}{}^{(k)} & C_{31}{}^{(k)} & C_{41}{}^{(k)} & C_{51}{}^{(k)} \\ C_{12}{}^{(k)} & C_{22}{}^{(k)} & C_{32}{}^{(k)} & C_{42}{}^{(k)} & C_{52}{}^{(k)} \\ C_{13}{}^{(k)} & C_{23}{}^{(k)} & C_{33}{}^{(k)} & C_{43}{}^{(k)} & C_{53}{}^{(k)} \\ C_{14}{}^{(k)} & C_{24}{}^{(k)} & M_{34}{}^{(k)} & C_{44}{}^{(k)} & C_{54}{}^{(k)} \\ C_{15}{}^{(k)} & C_{25}{}^{(k)} & M_{35}{}^{(k)} & C_{45}{}^{(k)} & C_{55}{}^{(k)} \end{pmatrix}$$

$$J_k{}^{-1} = \begin{pmatrix} \frac{C_{11}{}^{(k)}}{|J_k|} & \frac{C_{21}{}^{(k)}}{|J_k|} & \frac{C_{31}{}^{(k)}}{|J_k|} & \frac{C_{41}{}^{(k)}}{|J_k|} & \frac{C_{51}{}^{(k)}}{|J_k|} \\ \frac{C_{12}{}^{(k)}}{|J_k|} & \frac{C_{22}{}^{(k)}}{|J_k|} & \frac{C_{32}{}^{(k)}}{|J_k|} & \frac{C_{42}{}^{(k)}}{|J_k|} & \frac{C_{52}{}^{(k)}}{|J_k|} \\ \frac{C_{13}{}^{(k)}}{|J_k|} & \frac{C_{23}{}^{(k)}}{|J_k|} & \frac{C_{33}{}^{(k)}}{|J_k|} & \frac{C_{43}{}^{(k)}}{|J_k|} & \frac{C_{53}{}^{(k)}}{|J_k|} \\ \frac{C_{14}{}^{(k)}}{|J_k|} & \frac{C_{24}{}^{(k)}}{|J_k|} & \frac{C_{34}{}^{(k)}}{|J_k|} & \frac{C_{44}{}^{(k)}}{|J_k|} & \frac{C_{45}{}^{(k)}}{|J_k|} \\ \frac{C_{15}{}^{(k)}}{|J_k|} & \frac{C_{25}}{|J_k|} & \frac{C_{35}}{|J_k|} & \frac{C_{45}{}^{(k)}}{|J_k|} & \frac{C_{55}{}^{(k)}}{|J_k|} \end{pmatrix} \tag{17}$$

$$f_1{}^{(k)} = \Delta - \left(\beta I_k + \frac{\beta_B B_k}{K + B_k}\right) S_k + \theta R_k - \mu S_k$$
$$f_2{}^{(k)} = \left(\beta I_k + \frac{\beta_B B_k}{K + B_k}\right) S_k - (\alpha + \mu) E_k$$
$$f_3{}^{(k)} = \alpha E_k - (\gamma + d + \mu) I_k$$
$$f_4{}^{(k)} = \gamma I_k - (\theta + \mu) R_k$$
$$f_5{}^{(k)} = \psi I_k - (\in_2 + \in_3 - \in_1) B_k$$

∴ We can now write the method as:

$$\begin{pmatrix} S_{k+1} \\ E_{k+1} \\ I_{k+1} \\ R_{k+1} \\ B_{k+1} \end{pmatrix} = \begin{pmatrix} S_k \\ E_k \\ I_k \\ R_k \\ B_k \end{pmatrix} - \frac{1}{|J_k|}\begin{pmatrix} C_{11}{}^{(k)} & C_{21}{}^{(k)} & C_{31}{}^{(k)} & C_{41}{}^{(k)} & C_{51}{}^{(k)} \\ C_{12}{}^{(k)} & C_{22}{}^{(k)} & C_{32}{}^{(k)} & C_{42}{}^{(k)} & C_{52}{}^{(k)} \\ C_{13}{}^{(k)} & C_{23}{}^{(k)} & C_{33}{}^{(k)} & C_{43}{}^{(k)} & C_{53}{}^{(k)} \\ C_{14}{}^{(k)} & C_{24}{}^{(k)} & M_{34}{}^{(k)} & C_{44}{}^{(k)} & C_{54}{}^{(k)} \\ C_{15}{}^{(k)} & C_{25}{}^{(k)} & M_{35}{}^{(k)} & C_{45}{}^{(k)} & C_{55}{}^{(k)} \end{pmatrix}\begin{pmatrix} f_1{}^{(k)} \\ f_2{}^{(k)} \\ f_3{}^{(k)} \\ f_4{}^{(k)} \\ f_5{}^{(k)} \end{pmatrix} \tag{18}$$

$$\begin{pmatrix} S_{k+1} \\ E_{k+1} \\ I_{k+1} \\ R_{k+1} \\ B_{k+1} \end{pmatrix} = \begin{pmatrix} S_k \\ E_k \\ I_k \\ R_k \\ B_k \end{pmatrix} - \begin{pmatrix} \frac{C_{11}{}^{(k)}}{|J_k|} & \frac{C_{21}{}^{(k)}}{|J_k|} & \frac{C_{31}{}^{(k)}}{|J_k|} & \frac{C_{41}{}^{(k)}}{|J_k|} & \frac{C_{51}{}^{(k)}}{|J_k|} \\ \frac{C_{12}{}^{(k)}}{|J_k|} & \frac{C_{22}{}^{(k)}}{|J_k|} & \frac{C_{32}{}^{(k)}}{|J_k|} & \frac{C_{42}{}^{(k)}}{|J_k|} & \frac{C_{52}{}^{(k)}}{|J_k|} \\ \frac{C_{13}{}^{(k)}}{|J_k|} & \frac{C_{23}{}^{(k)}}{|J_k|} & \frac{C_{33}{}^{(k)}}{|J_k|} & \frac{C_{43}{}^{(k)}}{|J_k|} & \frac{C_{53}{}^{(k)}}{|J_k|} \\ \frac{C_{14}{}^{(k)}}{|J_k|} & \frac{C_{24}{}^{(k)}}{|J_k|} & \frac{C_{34}{}^{(k)}}{|J_k|} & \frac{C_{44}{}^{(k)}}{|J_k|} & \frac{C_{45}{}^{(k)}}{|J_k|} \\ \frac{C_{15}{}^{(k)}}{|J_k|} & \frac{C_{25}}{|J_k|} & \frac{C_{35}}{|J_k|} & \frac{C_{45}{}^{(k)}}{|J_k|} & \frac{C_{55}{}^{(k)}}{|J_k|} \end{pmatrix}\begin{pmatrix} f_1{}^{(k)} \\ f_2{}^{(k)} \\ f_3{}^{(k)} \\ f_4{}^{(k)} \\ f_5{}^{(k)} \end{pmatrix}$$

Numerical results and interpretation

Where $k = 0, 1, 2, 3, 4, \cdot \quad \cdot \quad \cdot$

First iteration

When $k = 0$,

Using $(S_0, E_0, I_0, R_0, B_0) = (10, 5, 3, 2, 300)$
$\alpha = 0.25,\ \mu = 0.025,\ d = 0.00008,\ \beta = 0.075,\ \theta = 0.5,\ \Lambda = 10,\ \mathrm{K} = 1000000,\ \psi = 1.5,$
$\gamma = 1.25,\ \ \beta_B = 0.00012,\ \epsilon_1 = 1.00,\ \epsilon_2 = 0.83,\ \epsilon_3 = 1.6$
$|J_k| = x_1x_5[\alpha(\beta x_6S_k + \psi x_7) - x_3x_4x_6] - \alpha x_2[\psi x_5x_7 + x_6(\beta x_6S_k - \theta\gamma)] = -0.0666$
$C_{11}{}^{(0)} = x_3x_4x_5x_6 - \alpha x_5(\beta x_6S_k + \psi x_7) = 0.1224$
$C_{12}{}^{(0)} = x_2x_4x_5x_6 = 0.2154$
$C_{13}{}^{(0)} = \alpha x_2x_5x_6 = 0.0422$
$C_{14}{}^{(0)} = \alpha\gamma x_2x_6 = 0.1005$
$C_{15}{}^{(0)} = \alpha\psi x_2x_5 = 0.0443$
$C_{21}{}^{(0)} = -\alpha[\psi x_5x_7 + x_6(\beta x_5S_k - \theta\gamma)] = 0.0827$
$C_{22}{}^{(0)} = x_1x_4x_5x_6 = 0.2393$
$C_{23}{}^{(0)} = \alpha x_1x_5x_6 = 0.0469$
$C_{24}{}^{(0)} = \alpha\gamma x_1x_6 = 0.1117$
$C_{25}{}^{(0)} = \alpha\psi x_1x_5 = 0.0492$
$C_{31}{}^{(0)} = \psi x_3x_5x_7 - x_3x_6(\beta x_5 - \theta\gamma) = 0.2303$
$C_{32}{}^{(0)} = x_1x_5(\beta x_6S_k + \psi x_7) - x_2\left(\psi x_5x_7 + x_6(\beta x_5S_k - \theta\gamma)\right) = 0.2152$
$C_{33}{}^{(0)} = x_1x_3x_5x_6 = 0.0516$
$C_{34}{}^{(0)} = \gamma x_1x_3x_6 = 0.1229$
$C_{35}{}^{(0)} = \psi x_1x_3x_5 = 0.0541$
$C_{41}{}^{(0)} = \theta[x_3x_4x_6 - \alpha(\beta x_6S_k - \psi x_7)] = 0.1167$
$C_{42}{}^{(0)} = \theta x_2x_4x_6 = 0.2051$
$C_{43}{}^{(0)} = \theta\alpha x_2x_6 = 0.0402$
$C_{44}{}^{(0)} = x_1x_3x_4x_6 - \alpha(\beta x_6S_k + \psi x_7)(x_2 - x_1) = 0.1187$
$C_{45}{}^{(0)} = \alpha\theta\psi x_2 = 0.0422$
$C_{51}{}^{(0)} = x_7[\alpha\theta\gamma - x_3x_4x_5] = -3.3388 \times 10^{-12}$
$C_{52}{}^{(0)} = x_4x_5x_7(x_1 - x_2) = 2.01 \times 10^{-12}$
$C_{53}{}^{(0)} = \alpha x_5x_7(x_1 - x_2) = 3.9352 \times 10^{-13}$
$C_{54}{}^{(0)} = \alpha\gamma x_7(x_1 - x_2) = 9.3695 \times 10^{-13}$
$C_{55}{}^{(0)} = x_1x_5(x_3x_4 - \alpha\beta S_k) + \alpha x_2(\beta x_5S_k - \theta\gamma) = 8.4052 \times 10^{-3}$

$$f_1{}^{(0)} = \Lambda - \left(\beta I_0 + \frac{\beta_B B}{K + B}\right)S_0 + \theta R_0 - \mu S_0$$
$$= 10 - \left(0.075 \times 3 + \frac{0.00012 \times 300}{1000000 + 300}\right) \times 10 + (0.5 \times 2) - (0.025 \times 10)$$
$$f_1{}^{(0)} = 8.5000$$
$$f_2{}^{(0)} = \left(\beta I_0 + \frac{\beta_B B}{K + B}\right)S_0 - (\alpha + \mu)E_0$$
$$= \left(0.075 \times 3 + \frac{0.00012 \times 300}{1000000 + 300}\right) \times 10 - (0.25 + 0.025) \times 5 = 0.8750$$
$$f_3{}^{(0)} = \alpha E_0 - (\gamma + d + \mu)I_0$$
$$= (0.25 \times 5) - (1.25 + 0.00008 + 0.025) \times 3 = -2.5752$$
$$f_4{}^{(0)} = \gamma I_0 - (\theta + \mu)R_0$$
$$= (1.25 \times 3) - (0.5 + 0.025) \times 2 = 2.7000$$
$$f_5{}^{(0)} = \psi I_0 - (\epsilon_2 + \epsilon_3 - \epsilon_1)B_0$$
$$= (1.5 \times 3) - (0.83 + 1.6 - 1.0) \times 3 = -424.5$$

We substitute the values in equation (19)

$$\begin{bmatrix} S_1 \\ E_1 \\ I_1 \\ R_1 \\ B_1 \end{bmatrix} = \begin{bmatrix} S_0 \\ E_0 \\ I_0 \\ R_0 \\ B_0 \end{bmatrix} - \frac{1}{|J_0|}\begin{bmatrix} C_{11}{}^{(0)} & C_{12}{}^{(0)} & C_{13}{}^{(0)} & C_{14}{}^{(0)} & C_{15}{}^{(0)} \\ C_{21}{}^{(0)} & C_{22}{}^{(0)} & C_{23}{}^{(0)} & C_{24}{}^{(0)} & C_{25}{}^{(0)} \\ C_{31}{}^{(0)} & C_{32}{}^{(0)} & C_{33}{}^{(0)} & C_{34}{}^{(0)} & C_{35}{}^{(0)} \\ C_{41}{}^{(0)} & C_{42}{}^{(0)} & C_{43}{}^{(0)} & C_{44}{}^{(0)} & C_{45}{}^{(0)} \\ C_{51}{}^{(0)} & C_{52}{}^{(0)} & C_{53}{}^{(0)} & C_{54}{}^{(0)} & C_{55}{}^{(0)} \end{bmatrix}\begin{bmatrix} f_1{}^{(0)} \\ f_2{}^{(0)} \\ f_3{}^{(0)} \\ f_4{}^{(0)} \\ f_5{}^{(0)} \end{bmatrix} \tag{19}$$

$$= \begin{bmatrix} 10 \\ 5 \\ 3 \\ 2 \\ 300 \end{bmatrix} - \left[\frac{1}{-0.0666}\begin{bmatrix} 0.1224 & 0.2154 & 0.0422 & 0.1005 & 0.0443 \\ 0.0827 & 0.2393 & 0.0469 & 0.1117 & 0.0492 \\ 0.2303 & 0.2152 & 0.0516 & 0.1229 & 0.0541 \\ 0.1167 & 0.2051 & 0.0402 & 0.1187 & 0.0422 \\ C_{51}{}^{(0)} & C_{52}{}^{(0)} & C_{53}{}^{(0)} & C_{54}{}^{(0)} & C_{55}{}^{(0)} \end{bmatrix}\begin{bmatrix} 8.5000 \\ 0.8750 \\ -2.5752 \\ 2.7000 \\ -424.50 \end{bmatrix}\right]$$

$C_{51}{}^{(0)} = -3.3388 \times 10^{-12}$, $C_{52}{}^{(0)} = 2.01 \times 10^{-12}$, $C_{53}{}^{(0)} = 3.9352 \times 10^{-13}$, $C_{52}{}^{(0)} = 9.3695 \times 10^{-13}$, $C_{52}{}^{(0)} = 8.4052 \times 10^{-3}$

$$\begin{bmatrix} S_1 \\ E_1 \\ I_1 \\ R_1 \\ B_1 \end{bmatrix} = \begin{bmatrix} 109.3558 \\ 174.7166 \\ 34.2560 \\ 81.5691 \\ 35.9329 \end{bmatrix}$$

Therefore, $S_1 = 109.3558$, $E_1 = 174.7166$, $I_1 = 34.2560$, $R_1 = 81.5691$, $B_1 = 35.9329$

We repeat the iteration until the system of non-linear equations converges using Matlab software.

Number of Iterations (k)	$S^{(k)}$	$E^{(k)}$	$I^{(k)}$	$R^{(k)}$	$B^{(k)}$
0	10	5	3	2	300
1	109.3558	174.7166	34.2560	81.5619	35.9329
2	−22.3923	253.9151	49.7842	118.5337	52.2211
3	15.0578	231.4026	45.3702	108.0243	47.5911
4	18.6670	229.2329	44.9448	107.0114	47.1449
5	18.7012	229.2124	44.9408	107.0019	47.1407
6	18.7012	229.2124	44.9408	107.0019	47.1407

Table 1. Numerical Results from Newton-Raphson Method.

From the table 1 above, it is observed that the susceptible, exposed, infected, recovered population increases until after the second iteration before it reduces to the point of convergence after fifth iteration while the bacteria population is alternating. The solution of the variables are:$(S = 18.70, E = 229.21, I = 44.94,\ R = 107.00, B = 47.14)$

Using Microsoft excel package to plot the three nonlinear equations we obtain Figure 1. The sky blue curve is for $S^{(k)}$, brown curve is for $E^{(k)}$, ash curve is for $I^{(k)}$, yellow curve is for $R^{(k)}$ and dark blue curve is for $B^{(k)}$. The point where the values of the variables of the nonlinear system of equation remains constant after a number of iterations is said to be the point of convergence.

ISSN: 4120 – 5671
Volume 12, Number 1, 2026
Advanced Journal of Physical Science Research

Journal Homepage: www.bwjournal.org

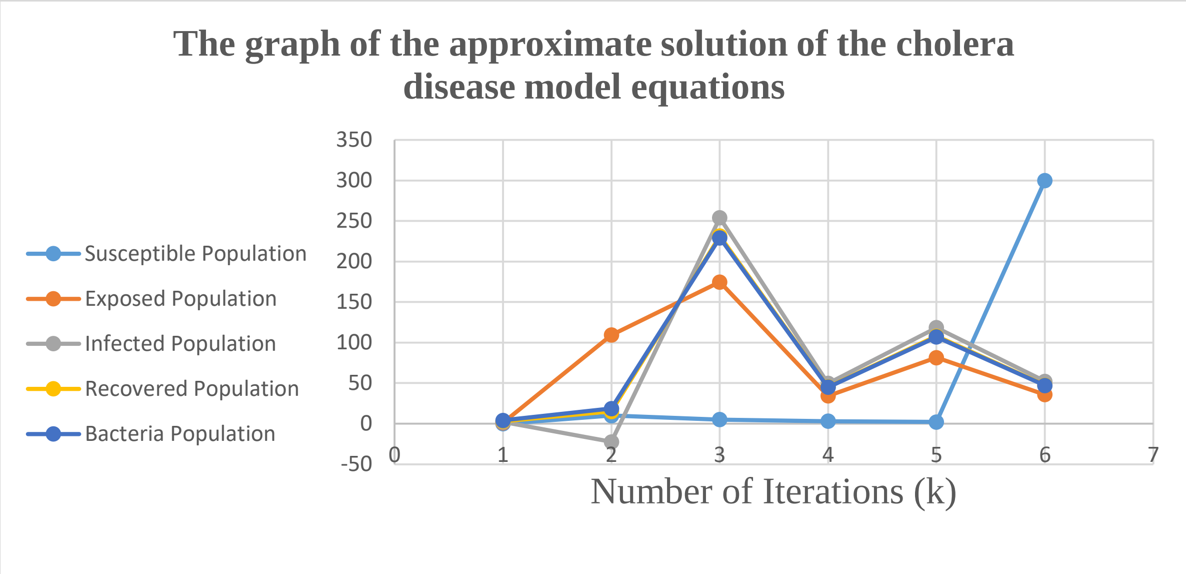

Figure 2: The graphical behavior of the numerical solution for the SEIRB Model

CONCLUSION

In this Paper, we formulated mathematical model equations of a cholera infection disease with the aid of the system of ordinary differential equations to study the convergence of the variables in the infectious diseases with five compartments; susceptible class (S), expose class (E), infected class (I), recovered class (R) and the bacteria population with their corresponding parameters using Newton–Raphson's iteration method. With this method, we observed that the variables in the system of equation converges after the sixth iteration. The results of the numerical solution (convergence) of the model system using MATLAB are ($S = 18.70, E = 229.21, I = 44.94$, $R = 107.00, B = 47.14$). The graph of the numerical simulation is plotted using excel software package and the graphical behaviour of the numerical solution is shown in figure 2 above.

REFERENCES

Manishkumar Jaiswal & Vinay Jadhav (2021). Numerical study of iterative methods for solving the non-linear equation. *International Journal of Creative research Thoughts (IJCRT)*, 9(10), 659 – 670.

Louis, E., Robert, K., & Thierry, L. (2005). Numerical methods for solving nonlinear equations. *John Wiley and sons Ltd.*

Francesco Casella & Bernhard Bachmann (2021). On the choice of initial guesses for the Newton-Raphson algorithm. *Applied Mathematics and Computation Volume 398.* https://doi.org/10.1016/j.amc.2021.125991.

Ebelechukwu, O. C. (2018). Comparison of Some Iterative Methods of Solving Nonlinear equations. *International Journal of Theoretical and Applied Mathematics*, 4(2), 22. Doi:10.11648/j.ijtam.20180402.11.

Hasan, A. (2016). Numerical Study of Some Iterative Methods for Solving Nonlinear equations. *International Journal of Engineering Science Invention*, 5(2), 01-10.

Moheuddin, M. M., Uddin, M. J., & Kowsher, M. (2019). A New Study to Find Out the Best Computational Method for Solving the Nonlinear Equation. Applied Mathematics and Sciences *An International Journal (MathSJ),* 6(3), 15-31. doi:10.5121/mathsj.2019.6302.

Grosan, C., & Abraham, A. (2008). A new approach for solving nonlinear equations systems. *IEEE Transactions on Systems, Man, and Cybernetics - Part A: Systems and Humans*, *38*(3), 698–714. https://doi.org/10.1109/TSMCA.2008.918599.

Hosseini, M. M., & Kafash, B. (2010). An efficient algorithm for solving system of nonlinear equations. *Applied Mathematical Sciences*, *4*(3), 119–131.

Montazeri, H., Soleymani, F., Shateyi, S., & Motsa, S. S. (2012). On a new method for computing the numerical solution of systems of nonlinear equations. *Journal of Applied Mathematics*, *2012*, 1–15. https:// doi.org/10.1155/2012/751975

Taheri, S., & Mammadov, M. (2012). Solving systems of nonlinear equations using a globally convergent optimization algorithm. *Global Journal of Technology & Optimization*, *3*, 132–138.