Chapter 11

Game Theory Practices in Business Organisation

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Introduction
Decisions are made under different conditions or situations. The process the decision maker takes absolutely depends on the situation in which he finds himself. Game theory is an operations research technique that deals with decision making under conflict or competition. A game consists of an infinite and finite number of players who can make strategic moves which will result into the best payoff of them. It represents a situation of lack of information in which intelligent opponents (players) are working in a conflicting environment. The game theory is a mathematical theory that with the general features of competitive situations where individuals or organizations with conflicting objective try to make decisions. Under this condition, a decision made by one decision maker affects the decision made by one or the other and the final outcome depends upon the decision of all the purities. This theory is applicable to wide variety of conditions such as two players struggling to win at monopoly game, chess, candidates, fighting to win an election, two enemies planning war tactics, firms struggling to gain market dominance, lunching advertisement campaigns by companies of the same product line, negotiation between organizations and unions are some of the conditions under which the theory of game can be applied.

Game theory is based on the minimax principle by J. Von Neuman in 1928 which implies that each competitions will act in order to minimize his minimum loss or maximize his minimum gain) or achieve best of the worst.

LIMITATION
1. Game theory does not describe now a game should e played, but only described the principles and procedure for selecting a play.
2. The application of game theory does not cover every condition and situations for decision making in an organization.
3. It s application and solution do not actually reflect a real life situation or problem.

CHARACTERISTICS OF GAMES
A game is competitive strategies that have the following characteristics
a. There are finite numbers of players or competitors. If the numbers of players are two, then it is known as two person game. While when player are more than two, it is called n-person game.
b. Every player have information about the possible course of action to be played.
c. Each player have information about all possible choices to be played by each player but do not know which to be chosen.
d. A play is said to occur when each play chooses a course of action available to him. Their chose are made at the same time by every players to avoid baize.
e. Every combination of courses of action results into outcome or payoff which may be positive, negative or zero. A negative payoff is known as loss.
f. The gain of a participant depends not only on his action but also on others.
g. The players make individual decision without informing other players.

GAME MODELS

Different kinds of game model are available based on the number of players, the sum of gain or loss, and the strategies to apply. Such as:

1. **NUMBER OR PERSON:** If a game is made of two players only, it is known as two-person game. If it is more than two players, it is called n-person game. An n-person game is always classified into n-mutually exclusive groups with all members of the group having the same interest.
2. **SUM OF PAYOFFS:** This means when the gain and loss of players is equal to zero. This is also known as zero-sum-gain or constant-sum game.
3. **NUMBER OF STRATEGIES:** This is when the number of strategies, moves or choices is finite of infinite; which can be known as finite game or infinite game.

BASIC CONCEPTS OF GAME THEORY

1. **GAME:** This is an activity that involves two or more persons. Actions by each player are regulated by rules which results in some gain, (Positive, negative) or zero by each player.
   A game determined by skill is known as game of strategy
   A game determined by chance is known as game of chance
   A finite 5game has a finite number of moves and choices, while an infinite game contains an infinite number of choices and moves.

2. **PLAYER:** Each participant or competitor of the game is called a player, who is a rational player.

3. **PLAY:** A play of a game occurs when each player chooses one courses of actions.

4. **STRATEGY:** This is regulated by sets of rules which a player chooses his course of action among the alternative courses of actions during the game. To decide a strategy, the player need not know the other players strategy.

5. **PURE STRATEGY:** This is a choose among alternative course of action that is strictly determined by the decision maker which is known as saddle point.

6. **MIXED STRATEGY:** This is when a game is not strictly determined and it does not have a saddle point. It is a strategy where courses of action are known to every player and the gain of a player is equal to the loss of the other player.

7. **OPTIMAL STRATEGY:** A strategy is puts the player in the most preferred position irrespective of the strategy of the other player is known as optimal strategy. Any deviation from this strategy would reduce his payoff.

8. **ZERO-SUM GAME:** This is determined by the total payoffs of all players at the end of the game. It is a situation whereby the total gain of player is equal to total loss of the other player which is known as maxi-min

9. **TWO-PERSON ZERO-SUM GAME:** This is also known as regular game or matrix game because the playoff matrix is rectangular. This game consists of two players where the gain of one player is equal to the loss of the other.

   Its characteristics consist of:
   (a) Only two player needed
   (b) A player have finite number of choice to take

(c) A particular strategy results in a payoff of gain or loss
(d) The total playoff at the end of a game is zero

10. **NON-ZERO-SUM GAME:** This is a situation where a third party for example a house receives or makes payment. The payoff matrix of this game, the left hand entries or a cell in the matrix tale are for a player and right hand entries in a cell are for the other player such as (2, 2; 6, -6) and (-6, 6; -2, -2). The play is done together, and the sums of payoffs are not equal to zero.

11. **PAYOFF:** This is the outcome or result of a particular game. Payoff may be positive or negative, gain or lose, and zero (fair).

12. **VARIABLES/ELEMENTS OF THE MATRIX:** It should be made clear that the figure in the matrix are representation of parties players opinion, ideas, decision and strategies that are represented with figure in order algebraically solve such problems

FORMULATION OF GAME MODEL

In the formulation of matrix game, data are required such as:

a. Two players A (maximizing) and B (minimizing)
b. Finite number of strategies choice moves
c. Finite payoff. A payoff is the result or outcome of interacting moves of a player.

		Player B				
		B_1	B_2	B_3	....	B_n
	A_1	a_{11}	a_{12}	a_{13}	....	a_{1n}
Player A	A_2	a_{21}	a_{22}	a_{23}	....	a_{2n}
	A_3	a_{31}	a_{32}	a_{33}	....	a_{3n}
		:	:	:	....	
		:	:	:	....	
	A_m	a_{m1}	a_{m2}	a_{m3}	....	a_{mn}

METHODS OF GAME THEORY

There are basically four methods used in solving game theory, but for the purposes of this stuck only three will be analyzed.

1. Pure strategy game
2. Mixed strategy game
3. Concept of Dominance
4. Sub games

PURE STRATEGY GAME

For game with pure strategy, you are required to determine the saddle point of the given game matrix. SADDLE POINT is the element of the game matrix which is the smallest in its row and the largest in its column.

ASSUMPTIONS OF PURE STRATEGY GAME

1. A game with a saddle point is said to be strictly determined
2. The row containing the saddle is the optimal strategy for the row player
3. The column containing the saddle point is the optimal strategy for the column player
4. The expected payoff (values of the game or outcome) to both players when the optimal strategy are adopted, it's also given by the saddle point
5. Expected value E(v)
   E(v) = + ve: means game is in favour of row player

E(v) = - ve: means game is in favour of column players
E(v) = 0: means a fair game, not in favour of both players

Example, solve the game matrix

	C_1	C_2	C_3	C_4	C5	Row min
R_1	10	8	-2	3	7	-2
R_2	8	7	6	2	9	2
R_3	7	6	8	(4)	10	4
R_4	10	-5	3	1	11	-5
R_5	6	4	8	3	12	3
Column max	10	8	8	(4)	12	

Saddle Point

In deriving the row minimum, the smallest element in the row matrix is selected for column maximum.
The saddle point is selected between the row minimum and the column maximum. The saddle point value is common to both and it's called the expected value. From the game matrix above, 4 is the saddle point and its positive (+ve) hence the E(v) is positive which means the game is in favour of row player.

DECISION RULE
Since the entry of 4 appears in both row and column, saddle point there for exists at R_3 C_4. The optimal strategy for row R_3 (x_4), and the optimal strategy for column player at C_4 (y_4)
Therefore, E(v) = 4 (value of the game
It positive hence the game is in favour of row player.

MIXED STRATEGIES
When a game is not strictly determined, then it has mixed strategies. That is optimal strategy is actually a combination of strategies. The basic idea is to develop a pattern of mixed strategies so that the expected gain or loss to a player should be the same regardless of the plan of the chance where in probabilities are assigned to the various strategies of the player.

MIXED STRATEGY GAME (2 X 2)
Algebraic and arithmetic methods are used for finding optimum strategies as well as game value for a 2 x 2 game

ALGEBRAIC METHOD OF 2 X 2 MATRIX

$$A = \begin{bmatrix} w & x \\ y & z \end{bmatrix}$$

This is a mixed strategy game if and only if!

1. W, z, > x and W, Z > Y or
2. X, y > W and X, y > Z

Otherwise, it is a pure strategy game. Hence, optimal strategy for row player is:

$$x_o = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$$

And optimal strategy for column player is:

$$y_o = \begin{bmatrix} q_1 & q_2 \end{bmatrix}$$

Where P_1 P_2, q_1 q_2 are frequency of players for row 1 and row 2, column 1 and column 2 respectively.

Therefore, the formulas are:

$$P_1 = \frac{z - y}{(w+z) - (x + y)} \quad \text{and } P_1 + P_2 = 1$$

$$q_1 = \frac{z - x}{(w+z) - (x + y)} \quad \text{and } q_1 + q_2 = 1$$

And,

$$E(v) = \frac{wz - xy}{(w+z) - (x + y)}$$

Example: Using algebraic method, solve the game matrix given:

$$A = \begin{bmatrix} 8 & -5 \\ 3 & 4 \end{bmatrix}$$

Solution

The conditions for carrying out our derivation are

1. 8, 4 > 3 and 4 > -5
2. -5, 3 > 8 and 3 > 4

If the matrix satisfied any of these conditions, then it is considered as a mixed strategy game

$$P_1 = \frac{4 - 3}{(8+4) - (-5 + 3)}$$

$$\frac{1}{12 - (-2)}$$

$$\frac{1}{12 + 2} = \frac{1}{14}$$

P_2

$$P_1 + P_2 = 1$$

$$\frac{1}{8} + P_2 = 1$$

$$P_2 = 1 - {}^1/_{14}$$

$$= \frac{13}{14}$$

Therefore:

$$x_0 = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} \quad \begin{bmatrix} {}^1/_{14} \\ {}^{13}/_{14} \end{bmatrix}$$

This is optimal strategy for row player

$$q_1 = \frac{4 - (-5)}{(8+4) - (-5 + 3)}$$

$$\frac{4 + 5}{12 - (-2)} = \frac{9}{12+2} = \frac{9}{14}$$

$$q_1 + q_2 = 1$$

$$\frac{9}{14} + q_2 = 1$$

$$q_2 = 1 - {}^9/_{14} = {}^5/_{14}$$

Therefore,

$$y_o = \|q_1, q_2\| = \|{}^9/_{14}, {}^5/_{14}\|$$

This is an optimal strategy for column player

EXPECTED VALUE E (v)
The expected of the mixed game matrix is derived using the formula

$$E(v) = \frac{wz - xy}{(w+z) - (x+y)}$$
$$\frac{8(4) - -5(3)}{(8+4) - (-5+3)}$$
$$\frac{32 - (-15)}{12 - (-2)}$$
$$\frac{32 - 15}{12 + 2}$$
$$\frac{47}{14}$$

Therefore E(v) is +ve. The game is in favour of row player

CONCEPT OF DOMINANCE FOR
UNDIAGONAL MATRIX
If a matrix is given that is not diagonal, what is expected to be done is to convert the matrix into a diagonal matrix of 2 x 2 before applying a formula. It is the conversion that is known as concept of dominance. The matrix could be given in 3 x 5, 4 x 5, 3 x 4 etc, and it should be converted to 2 x 2.

Since formula exists for the solution of 2 x 2 matrix only, it is better to reduce an m x n mixed strategy game matrix to 2 x 2, using the concept of dominance.

m x n DOMINANCE MODEL

$$Z \begin{vmatrix} z_{11} & z_{12} & z_{13} & \ldots\ldots\ldots & z_{1n} \\ z_{21} & z_{22} & z_{23} & \ldots\ldots\ldots & z_{2n} \\ z_{31} & z_{32} & z_{33} & \ldots\ldots\ldots & z_{3n} \\ : & : & : & \ldots\ldots\ldots & \\ : & : & : & \ldots\ldots\ldots & \\ z_{m1} & za_{m2} & z_{m3} & \ldots\ldots\ldots & z_{mn} \end{vmatrix}$$

1. If $a_{ij} \geq a_{kj}$, where j = 1, 2, 3 ………… n. Then the ith row is said to dominate the kth row which is regarded as recessive row.
2. If $a_{ij} \leq a_{ik}$, where i = 1, 2, 3 ………… m. Then the ith column is said to dominate the kth column which is regarded as recessive column.

ALSO NOTE
1. For optimization purposes, a player will never wish to play a recessive strategy.
2. Recessive rows and columns may therefore be omitted from the game matrix.

3. The concept of dominance is used to reduce an m x n matrix into 2 x 2 matrix.
4. The optimal strategy for the 2 x 2 matrix game is translated into optimal strategies for the original m x n matrix by inserting zeros (0) frequencies in the recessive rows and columns of the strategy vector or with the value of the optimal value obtained.
5. The value of the 2 x 2 matrix is also the value of the original m x n game matrix.

Example: Solve the game matrix given using Dominance

$$A = \begin{vmatrix} -6 & 0 & 3 & 4 \\ 3 & 5 & -2 & 6 \\ 2 & 4 & 6 & 1 \\ 1 & 3 & 2 & -4 \\ 4 & -5 & 7 & 0 \end{vmatrix}$$

Solution: The matrix is given in 5 x 4 matrix, also not that m is row and n is column.

In order to reduce the 5 x 4 matrix into 2 x 2 matrix, find the total of all elements in each row and column. From the total row max, use the highest members of rows to eliminate or dominate rows whose number is smaller either below or above by deleting with a cross line. While in total column min, use the smallest numbers of columns to eliminate or dominate columns whose numbers are higher either proceeding or before by deleting with a cross line.

					Total Row max
	~~-6~~	~~0~~	~~3~~	~~4~~	~~1~~
	3	~~5~~	~~-2~~	6	12
	2	~~4~~	~~6~~	1	13
	~~1~~	~~3~~	~~2~~	~~-4~~	~~2~~
	~~4~~	~~-5~~	~~7~~	~~0~~	~~6~~
Total column min	4	~~7~~	~~16~~	7	

$$A = \begin{bmatrix} 3 & 6 \\ 2 & 1 \end{bmatrix}$$

Applying the Formula

$$P_1 = \frac{z - y}{(w+z) - (x + y)}$$

Substitute the matrix values into the formula

$$P_2 = \frac{1 - 2}{(3+1) - (6+2)} = \frac{-1}{4+8} = \frac{-1}{12}$$

$$P_2 = P_1 + P_2 = 1$$

$$1 - P_1 = P_2$$

$$P = 1 - {}^{-1}/_{12}$$

$$1 - {}^{1}/_{12} = {}^{11}/_{12}$$

Therefore,

$$x_o = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} {}^{-1}/_{12} \\ {}^{11}/_{12} \end{bmatrix}$$

Hence, replace the recessive rows with zeros (0)

$$x_o = \begin{bmatrix} x_1 \\ x_2 \\ \\ \\ \end{bmatrix} \quad \begin{bmatrix} 0 \\ {}^{1}/_{12} \\ \\ \\ \end{bmatrix}$$

$x_3 = {}^{11}/_{12}$
$x_4 \quad 0$
$x_5 \quad 0$

This is the optimal strategy for row player

$$q_1 = \frac{z - x}{(w+z) - (x+y)}$$

$$\frac{1 - 6}{(3+1) - (6+2)} = \frac{-5}{4+8} = \frac{-5}{12}$$

$q2 = q_1 + q_2 = 1$

$1 - q_2 = q_2$

$1 - {}^{5}/_{12} \quad = 1 - {}^{5}/_{12} \quad = \quad {}^{7}/_{12}$

Therefore,

$y_o = \left[q_1, q_2 \right] = \left[{}^{-5}/_{12}, {}^{7}/_{12} \right]$

Hence, replace recessive columns with zeros (0)

$y_o = \left[q_1, q_2, q_3, q_4, q_5 \right] = \left[{}^{-5}/_{12}, 0, 0, {}^{7}/_{12} \right]$

This is the optimal strategy for column players

EXPECTED VALUE E (v)

$$E(v) = \frac{wz - xy}{(w+z) - (x+y)}$$

$$\frac{3(1) - 6(2)}{(3+1) - (6+2)}$$

$$\frac{3 - 12}{3 - 8}$$

$$\frac{9}{-5} \quad \text{-ve}$$

The E(v) is negative, and it is in favour of column player.

Exercises

1. For any 2 x 2 two-person zero-sum game without any saddle paint, having payoff matrix for payer A as

Player A	Player B B_1	B_2
A_1	a_{11}	a_{12}
A_2	a_{21}	a_{22}

Find the optimal strategy and expected value of the game.

2. The payoff matrix of a game is given below. Find the solution of the game to A and B

		B			
	1	2	3	4	5
A 1	-4	-2	-2	3	1

2	1	0	-1	0	0
3	-6	-5	-1	-4	4
4	3	1	-6	0	-8

3. Find the saddle point and values of the following game

		Z			
		w	x	y	z
y	a	9	12	18	8
	b	15	-4	2	9
	c	11	12	13	10

4. A duo poly can be viewed as a game between two firms-suppose the profit to firm A for high or low prices is given as follows

		B	
		High	Low
	High	+10	+3
A	Low	+15	+5

a. What pricing strategy will each firm follow?
b. Is there any incentive for deceit? How many price fixing agreement?
c. Is this a zero-sum game when profit is used as the payoff?

5. Ukari's company is currently involved in negotiation with its union on the upcoming wage contract. Positive signs represent wage increase while negative signs represents wage reduction. What are the optimal strategies for the company as well as the union? What is the game value?

		Union			
		U_1	U_2	U_3	U_4
	C1	+0.25	+0.28	+0.14	-0.16
Company	C2	+0.20	+0.20	-0.20	+0.16
	C3	+0.14	+0.22	+0.18	+0.15
	C4	+0.25	+0.13	+0.10	-0.9

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